SEMESTRAL EXAMINATION
B. MATH III YEAR, II SEMESTER 2008-2009
INTRODUCTION TO STOCHASTIC PROCESSES

Answer as many questions as you can. The maximum you can score is 100.
Time limit: 3hrs

HMC stands for homogeneous Markov chain.

1. If $\{X_n\}$ is a HMC prove that
$P\{X_{n-2} = i_{n-2}, X_{n-1} = i_{n-1} | X_n = i_n, X_{n+1} = i_{n+1}\} = P\{X_{n-2} = i_{n-2}, X_{n-1} = i_{n-1} | X_n = i_n\}$ and find this conditional probabilty in terms of the transition matrix and the distribution of X_{n-2}. [15]

2.
a) Let $\{X_n\}$ be a Galton-Watson process with offspring distribution $\{p_k\}$. Find the transition probability matrix of this HMC.

[10]

b) Let $p_k = \frac{2}{3^{k+1}}(k \geq 0)$. Find the fixed points of the function $f(s) = \sum_{k=0}^{\infty} p_k s^k$ in $[0,1]$ and the extinction probability of the Galton watson process with offspring distribution $\{p_k\}$. [15]

3. Consider a two type Galton Watson process with mean matrix $\begin{bmatrix} 0.11 & 0.03 \\ 0.07 & 0.02 \end{bmatrix}$.
Find the extinction probabilities. [15]

4. Let $\{X_n\}$ be a HMC with transition matrix $P = ((p_{ij}))$ where $(p_{11}, p_{12}, p_{13}, ...) = (1, 0, 0, ...), (p_{21}, p_{22}, p_{23}, ...) = (0, 1/2, 1/2, 0, 0, ...)$,

and, for $i \geq 3, p_{ij} = \begin{cases} \frac{1}{2^{i-2}} \text{ if } j = 2 \\ 1 - \frac{1}{2^{i-2}} \text{ if } j = i+1 \\ 0 \text{ if } j \neq 2 \text{ and } j \neq i+1 \end{cases}$.

Find all the stationary distributions. [20]

5. Let $\{X_t\}$ be a birth and death process with state space $\{0,1\}$ and birth and death rates $\lambda_0 = 1, \mu_1 = 3$. Find $P\{X_t = 0 | X_0 = 1\}$. [25]

6. Let $\{X_t\}$ be a Piosson process with parameter λ. Prove that

$\lim_{s \to o} \frac{P\{X_{t+s} - X_t \geq 2\}}{s^2}$ exists and find the value of this limit. [10]